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Choke pressure in pipeline restrictions

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Abstract

A simple analytical formula is derived for the choke pressure in a pipeline restriction discharging gas. This differs from the standard formula based on ideal nozzle flow in that lower choke pressures are predicted. The formula may be applied directly to safety valves and is particularly recommended for valves with low vapour discharge coefficients.

Keywords: Choke pressure; Pipeline restriction; Safety valve; Discharge coefficient; Compressible fluid

1. Introduction

The design of a pipeline system for compressible fluid flow involves consideration of the phenomenon of choked flow for which the mass flowrate achieves a maximum value. Choked flow may occur at a change of flow area, e.g. at a pipe enlargement or in a component such as a control or safety valve, where there is an area reduction. A requirement for choked flow is that a certain minimum pressure (the choke pressure) is attained at the minimum flow area; this in turn depends on the flow conditions downstream being such as to create a sufficiently low backpressure at the potential choke location, i.e. a backpressure less than or equal to the choke pressure.

In the case of emergency pressure relief system design [1,2], one of the many research requirements is a need for improved understanding of the flow and operational characteristics of safety valves. This provided the stimulus for the present work which is focussed on choked gas flow through pipeline restrictions, a particular example of which is the safety valve. The objective is to derive a simple formula for the actual choke

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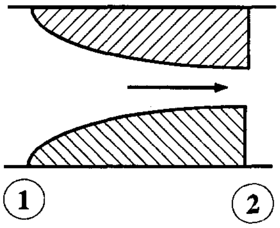

Fig. 1. Model of a pipeline restriction.

pressure (as opposed to the familiar formula based on ideal flow) that takes into account the irreversible losses incurred in the restriction.

Fig. 1 shows a nozzle installed in a pipe, an arrangement often used to model pipeline restrictions such as control and safety valves. In the case of choked flow through an *ideal* nozzle for which the thermodynamic flowpath is isentropic and described by:

$$pv^{\gamma} = \text{const} \tag{1}$$

the mass flowrate and pressure ratio are given by [3]:

$$\dot{M}_{c,id} = A_2\, p_{t1} \sqrt{\frac{\gamma \tilde{M}}{Z R_o T_{t1}}}\; r_{c,id}^{\frac{\gamma+1}{2\gamma}} \tag{2}$$

and

$$r_{c,id} = p_{c,id}/p_{t1} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \tag{3}$$

These equations may be applied to non-ideal gases ($pv = ZR_oT/\tilde{M}$) under the usual approximation of constant compressibility factor Z.

On the other hand, flow through a non-ideal nozzle is characterized by irreversible energy losses due to friction and form drag. These losses effectively reduce the flow capacity of the nozzle in comparison with the ideal case. A discharge coefficient C_D is then defined in order to relate the actual mass flowrate $\dot{M}_c$ and the ideal mass flowrate $\dot{M}_{c,id}$ as follows,

$$\dot{M}_c = C_D \dot{M}_{c,id} \tag{4}$$

The usual means of determining C_D is by experimental measurement of $\dot{M}_c$ and subsequent comparison with $\dot{M}_{c,id}$ as calculated from Eq. (2).

Due to irreversible losses, the flow in a non-ideal nozzle is no longer isentropic and so cannot be described thermodynamically by Eq. (1). This then means that Eq. (3), which is based on the validity of Eq. (1), cannot correctly describe the actual choke pressure ratio in a non-ideal nozzle. Eq. (3) may still be used in Eq. (2), however, since this is only for purposes of determining $\dot{M}_{c,id}$ as a reference flowrate.

The purpose of the present work is to determine the actual choke pressure ratio r_c for a non-ideal nozzle that takes account of the inherent losses.

2. Analysis

Although the flow through a non-ideal nozzle is not isentropic, it is reasonable to assume that it is adiabatic, i.e. the total temperature [3]

$$T_t = T_s\left[1 + \frac{1}{2}(\gamma - 1)Ma^2\right] \tag{5}$$

is constant throughout the flowpath. In particular, at the nozzle throat (location 2 in Fig. 1) where $Ma_2 = 1$, the total and static temperatures are related by:

$$T_{t2} = \frac{1}{2}(\gamma + 1)T_{s2} \tag{6}$$

and also $T_{t2} = T_{t1}$ as required by adiabaticity.

By definition, the Mach number, Ma, at any location in the nozzle is:

$$Ma = u\sqrt{\frac{\tilde{M}}{\gamma Z R_o T_s}} \tag{7}$$

and it follows that the choke velocity at the nozzle throat (where $Ma_2 = 1$) is given by:

$$u_2 = \sqrt{\frac{\gamma Z R_o T_{s2}}{\tilde{M}}} \tag{8}$$

Now, from mass continuity considerations,

$$\dot{M}_c = \rho_2 A_2 u_2 \tag{9}$$

and for a non-ideal gas,

$$\rho_2 = \frac{p_{s2}\tilde{M}}{Z R_o T_{s2}} \tag{10}$$

Combining Eqs. (8)–(10) gives the following expression for the choking mass flowrate:

$$\dot{M}_c = A_2 p_{s2}\sqrt{\frac{\gamma\tilde{M}}{Z R_o T_{s2}}} \tag{11}$$

However, $\dot{M}_c$ is also given by Eqs. (2)–(4), i.e.

$$\dot{M}_c = C_D A_2 p_{t1}\sqrt{\frac{\gamma\tilde{M}}{Z R_o T_{t1}}}\, r_{c,id}^{\frac{\gamma+1}{2\gamma}} \tag{12}$$

and equating the right-hand side with that of Eq. (11) gives

$$r_c = p_c/p_{t1} = C_D r_{c,id}^{\frac{\gamma+1}{2\gamma}}\sqrt{\frac{T_{s2}}{T_{t1}}} \tag{13}$$

Table 1
Safety valve discharge coefficients (vapour/gas)

C_D range	0.95–1.0	0.90–0.95	0.85–0.90	0.80–0.85	0.671–0.80
No. of valve series in C_D range	21	24	8	5	4

where r_c is the *actual* choke pressure ratio. Now, since

$$\sqrt{\frac{T_{s2}}{T_{t1}}} = \sqrt{\frac{T_{s2}}{T_{t2}}} = \sqrt{\frac{2}{\gamma+1}} = r_{c,id}^{\frac{\gamma-1}{2\gamma}} \tag{14}$$

Eq. (13) reduces to the simple form:

$$r_c = C_D r_{c,id} \tag{15}$$

which is the required result of this paper.

In the case of safety valves, the discharge coefficients of most available types can be found in the Red Book [4]. A study of these discharge coefficients reveals magnitudes in the range $0.671 \leq C_D \leq 0.975$ which may be broken down into sub-ranges as shown in Table 1 for a total of 62 series of safety valves. Thus, 73% of safety valves have $0.90 \leq C_D < 1.0$ while 27% have $0.671 \leq C_D < 0.90$.

Eq. (15) then shows that the actual choke pressure may be significantly less than that calculated from Eq. (3). Thus, in theory at least, a safety valve will operate in the choked flow mode providing that the backpressure on the valve $p_{back} < p_c$ where, from Eq. (15),

$$p_c = C_D p_{c,id} \tag{16}$$

This is a more stringent requirement than that often quoted, namely that choked flow prevails for $p_{back} < p_{c,id}$.

In practice, however, good design of the valve tailpipe will ensure that the backpressure, p_{back}, is much less than the choke pressure, p_c, over most of the venting transient. This is normally assured providing that the familiar rules [5] governing backpressure limitations are observed.

3. Conclusions

A simple formula has been derived for the actual choke pressure ratio of flow through a pipeline restriction such as a safety valve. This is given by Eq. (15) and shows that, in the range $0.671 \leq C_D \leq 0.975$, choke pressures can be 2.5%–33% lower than those predicted by the familiar ideal nozzle formula (Eq. (3)). This finding need not raise doubts that existing safety valve installations will not perform as designed provided that existing rules on backpressure limitation have been observed.

4. Notation

A_2 Throat area (m^2)
C_D Discharge coefficient
Ma Mach number

Ma_2 Mach number at nozzle throat
$\dot{M}_c$ Choking mass flowrate ($kg\,s^{-1}$)
$\dot{M}_{c,id}$ Ideal choking mass flowrate ($kg\,s^{-1}$)
$\tilde{M}$ Molar mass or molecular weight ($kg\,kmol^{-1}$)
p Pressure (Pa)
p_{atmos} Atmospheric pressure (Pa)
p_c Choke pressure: value of p_{s2} when $Ma_2 = 1$ (Pa)
$p_{c,id}$ Ideal choke pressure (Pa)
p_{back} Backpressure on valve (Pa)
p_{s1} Static pressure at nozzle inlet (Pa)
p_{s2} Static pressure at nozzle throat (Pa)
p_{t1} Total pressure at nozzle inlet (Pa)
R_o Universal gas constant (8314 $J\,kmol^{-1}\,K$)
r_cChoke pressure ratio (p_c/p_{t1})
$r_{c,id}$ Ideal choke pressure ratio
T_s Static temperature (K)
T_{s2} Static temperature at nozzle throat (K)
T_{t1} Total temperature at nozzle inlet (K)
T_{t2} Total temperature at nozzle throat (K)
u Gas velocity ($m\,s^{-1}$)
u_2 Choke velocity ($m\,s^{-1}$)
v Specific volume ($m^3\,kg^{-1}$)
Z Compressibility factor for non-ideal gas
ρ_2 Gas density at nozzle throat ($kg\,m^{-3}$)
γ Ratio of specific heats (c_p/c_v)

References

[1] H.G. Fisher et al., Emergency Relief System Design Using DIERS Technology (DIERS Project Manual), AIChE, 345 East 47th Street, New York 10017, 1992, ISBN 0-8169-0568-1.
[2] A. Benuzzi and J.M. Zaldivar (Eds.), Safety of Chemical Batch Reactors and Storage Tanks, Kluwer Academic Publishers Group, P.O. Box 322, 3300 AH Dordrecht, The Netherlands, 1991, ISBN 0-7923-1233-3.
[3] A.H. Shapiro, The Dynamics and Thermodynamics of Compressible Fluid Flow, Vol. 1, John Wiley and Sons, 1953.
[4] Pressure Relief Device Certifications 1994, The National Board of Boiler and Pressure Vessel Inspectors, 1055 Crupper Avenue, Columbus, Ohio 43229-1183, USA.
[5] API RP 520. Sizing, Selection and Installation of Pressure-Relieving Devices in Refineries, Part 1—Sizing and Selection, 6th edn., March 1993.